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FACULTY OF ENGINEERING AND ARCHITECTURE
MECH 230 - DYNAMICS – QUIZ 1 – FALL 2013

NAME: **ID No:** **Oct. 12, 2013**

1.5 hours closed book quiz

- Solve on the answer booklet in sequence.
- Question sheet will not be corrected and must be returned.
- Write clearly. Clarity is important in grading.
- Vectors are indicated in bold.
- Take g=9.81 m/s^2.

print use eqn.

No.1 – (40%)

The y-coordinate of a particle in curvilinear motion is given by $y=4t^3 - 3t$, where y is in meters and t in seconds. Also, the particle has an acceleration in the x-direction $a_x= 12t$ m/s^2. At t=0s the particle is at the origin (0,0) and its velocity in the x-direction is 4 m/s.

5% 1- Write an expression of the position **r**, the velocity **v** and the acceleration **a** as a function of time t in the x-y frame of reference.

5% 2- Determine the velocity of the particle at t=1s. in Cartesian form

5% 3- Determine the acceleration of the particle at t=1s.

5% 4- Determine the position of the particle at t=1s.

5% 5- Construct the position **r**, velocity **v**, and acceleration **a** in the x-y frame of reference at t=1s.

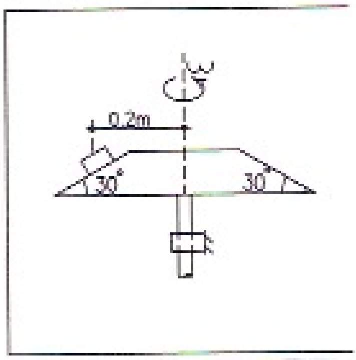
5% 6- Determine the normal and tangential components of acceleration of the particle at t= 1s. Show them on the graph of question 5.

5% 7- Determine the radial and transverse components of acceleration of the particle at t= 1s. Show them on the graph of question 5. (draw the same excluding t,n components)

5% 8- Determine r, $\dot{r}$, θ, $\dot{\theta}$, and $\ddot{\theta}$.

No.2 – (20%)

The small object of mass 1 kg is placed on the rotating conical surface at the radius shown. The coefficient of static friction between the object and the rotating surface is 0.8. Assume very gradual angular velocity changes.

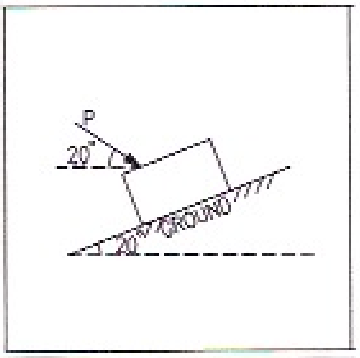


10% 1- Draw the free body diagram and the kinetic diagram of the object in the plane of the motion.

10% 2- Determine the maximum velocity of the object and consequently the maximum angular velocity $\dot{\theta}$ of the cone about the vertical axis for which the object will not slip.

No.3 – (20%)

The 20-kg package is at rest when on an incline when a force P is applied to it. Starting from rest at t=0 s, 10 seconds is required for the package to travel 5 m up the incline. The static and kinetic coefficients of friction between the package and the incline are both equal to 0.3.

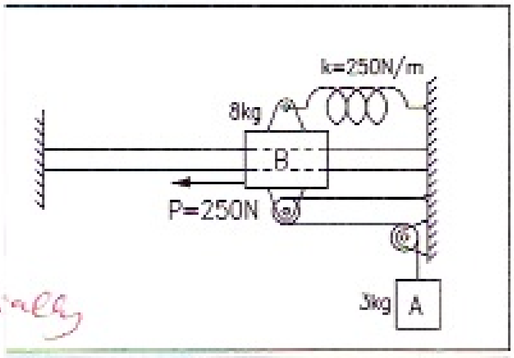


10% 1- Draw the free body diagram and the kinetic diagram of the package.

10% 1- Determine the magnitude of the force P.

No.4 – (20%)

The system shown is at rest when a constant 250 N force is applied to collar B. Collar B has a mass of 8 kg and moves on a frictionless horizontal rod. Block A has a mass of 3 kg and moves in the vertical plane. The attached horizontal spring to collar B has a stiffness of 100 N/m and is unstretched in the shown position. The system starts its motion from rest.



10% 1- Draw the free body diagram of collar B and block A separately.

10% 1- Determine the speed of collar B and block A when B has 0.4 m.

Date : FALL 2013 october 12, 2013	Name : QUIZ1 SOLUTION	Course : MECH 230

ON MY HONOR, I WILL NOT GIVE OR RECEIVE
ANY ASSISTANCE ON THIS QUIZ OR EXAM.

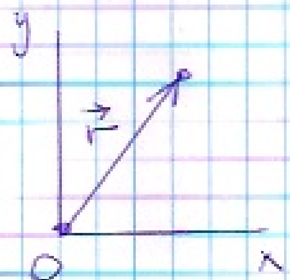
Signature: ____________________

PROBLEM 1 40% @ 5 Pts

Given: $y = 4t^3 - 3t$; $y \doteq m$; $t \doteq s$

@ $t = 0$; $(x_0 = 0 ; y_0 = 0)$ $\quad a_x = 12t \ m/s^2$

$v_{x_0} = 4 \ m/s$



1_ Find $\vec{r}$, $\vec{V}$, $\vec{a}$ as f(t) ; $\vec{r} = x(t)\hat{i} + y(t)\hat{j}$

$a_x = 12t$; (a_x, t, V_x) ; $a_x = \frac{dV_x}{dt}$

$$\int_4^{V_x} dV_x = 12\int_0^t t\,dt$$

$V_x - 4 = \frac{12}{2}t^2$; $V_x = 4 + 6t^2$; (V_x, x, t)

⇒ use $V_x = \frac{dx}{dt}$; $\int_0^t (4 + 6t^2)\,dt = \int_0^x dx$

$X = 4t + \frac{6t^3}{3}$; $X = 4t + 2t^3$

$\therefore \vec{r} = (4t + 2t^3)\hat{i} + (4t^3 - 3t)\hat{j}$ m Ans.

$\vec{V} = V_x\hat{i} + V_y\hat{j}$; $V_y = 12t^2 - 3$; $a_y = 24t$

$\vec{V} = (4 + 6t^2)\hat{i} + (12t^2 - 3)\hat{j}$ m/s Ans.

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$\vec{a} = (12t)\hat{i} + (24t)\hat{j}$ m/s² Ans.

2_ Find $\vec{V}$ @ t = 1s ; $\vec{V} = 10\hat{i} + 9\hat{j}$ m/s Ans.

3_ Find $\vec{a}$ @ t = 1s ; $\vec{a} = 12\hat{i} + 24\hat{j}$ m/s² Ans.

4_ Find $\vec{r}$ @ t = 1s ; $\vec{r} = 6\hat{i} + \hat{j}$ m Ans.

5- Construct $\vec{r}, \vec{v}, \vec{a}$ @ t=1s.

$\theta' = \tan^{-1}\left(\frac{9}{10}\right) = 41.99°$

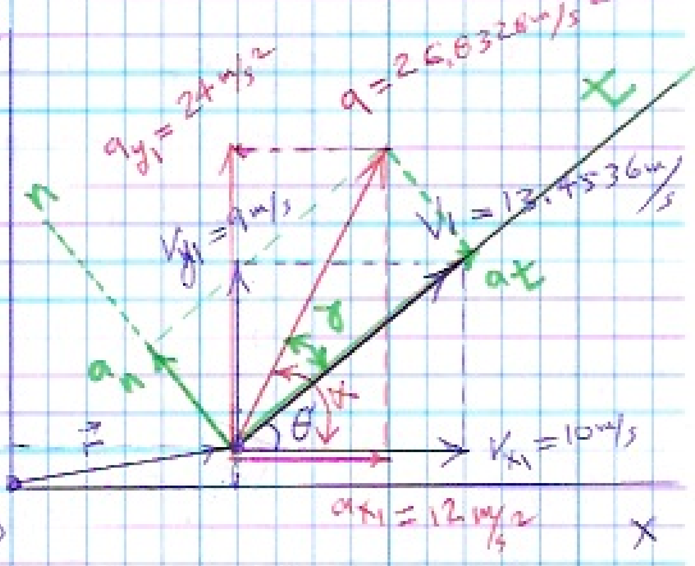
$V_1 = \sqrt{10^2+9^2} = 13.4536\ m/s$

$a = \sqrt{12^2+24^2} = 26.8328\ m/s^2$

$\alpha = \tan^{-1}\left(\frac{24}{12}\right) = 63.43°$

$\gamma = \alpha - \theta' = 63.43° - 41.99°$

$= 21.44°$



6- Find a_n, a_t @ t=1s & draw

$a_t = a \cos\gamma$

$a_t = 26.8328 \cos 21.44° = 24.976\ m/s^2$ Ans.

$a_n = a \sin\gamma = 26.8328 \sin 21.44° = 9.808\ m/s^2$ ans.

7- Find a_r & a_θ @ t=1s

&8-

$r = \sqrt{6^2+1^2} = 6.0828\ m$

$\theta = \tan^{-1}\left(\frac{1}{6}\right) = 9.46°$

$\vec{r} = r\hat{u}_r = 6.0828\hat{u}_r\ m$

$\beta = \theta' - \theta = 41.99° - 9.46°$

$\beta = 32.53°$

$v_r = 13.4536 \cos(32.53°) = 11.3429\ m/s = \dot{r}$

$v_\theta = 13.4536 \sin 32.53° = 7.2346\ m/s = r\dot{\theta}$

$\therefore \dot{\theta} = \frac{7.2346}{6.0828} = 1.1893\ rad/s$

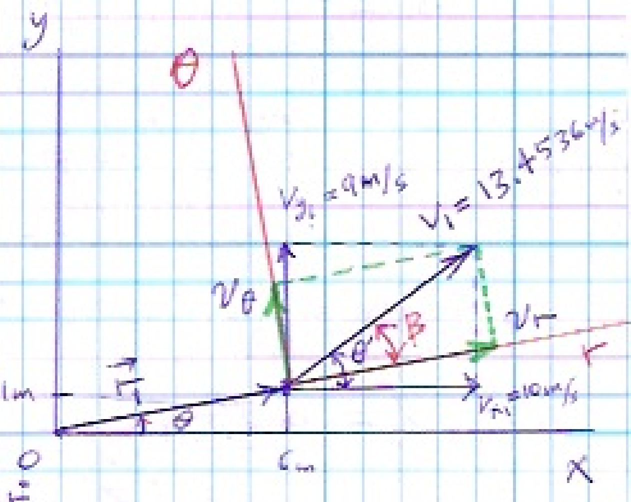
$\epsilon = \alpha - \theta = 63.43° - 9.46° = 53.97°$

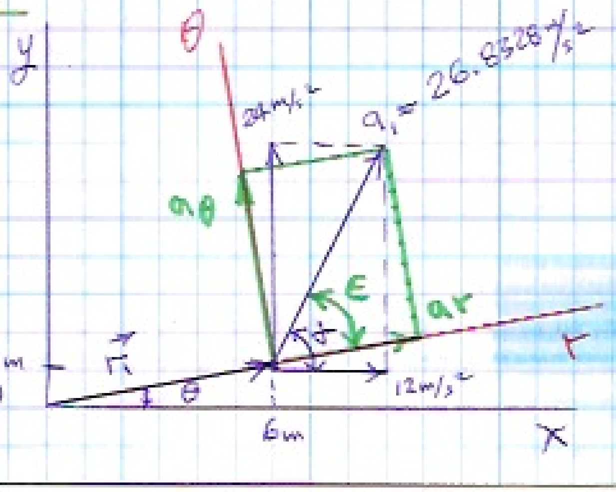
$a_r = 26.8328 \cos 53.97° = 15.7833\ m/s^2$ Ans.

$a_\theta = 26.8328 \sin 53.97° = 21.6999\ m/s^2$ Ans.

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$; $21.6999 = 6.0828\ddot{\theta} + 2(11.3429)(1.1893)$

$\ddot{\theta} = -0.8681\ rad/s^2$ Ans.



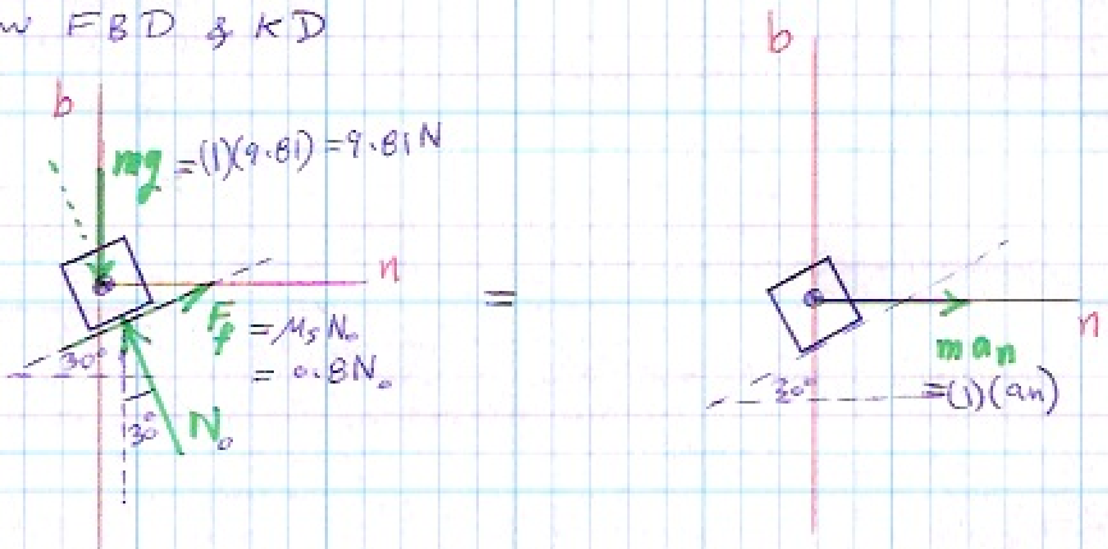


PROBLEM 2: 20%

Given: $m = 1 kg$; $\mu_s = 0.8$

Very gradual angular velocity changes ⇒ $a_t = 0$.

10% 1- Draw FBD & KD



2- For object not to slip ⇒ $F_f = \mu_s N_o$ (impending slipping)

eqs. of motion:

$+\uparrow \Sigma F_b = 0$; $N_o \cos 30° - 9.81^N + 0.8 N_o \sin 30° = 0$ ---(1)

$\overset{+}{\rightarrow} \Sigma F_n = ma_n$; $0.8 N_o \cos 30° - N_o \sin 30° = (1) a_n$ ---(2)

eqn. (1) ⇒ $N_o(\cos 30° + 0.8 \sin 30°) = 9.81^N$

$N_o = 7.7487 N$

eqn. (2) ⇒ $N_o(0.8 \cos 30° - \sin 30°) = a_n$; $a_n = 1.4941 m/s^2$

$a_n = \frac{v^2}{R}$ ⇒ $1.4941 = \frac{v^2}{0.2}$; $v^2 = 0.2988$

$V_{max} = 0.5466 m/s$ Ans.

$V = r\dot{\theta}$; $\dot{\theta}_{max} = \frac{V}{r} = \frac{0.5466}{0.2} = 2.7332 rad/s$ Ans.

PROBLEM 3 : 20%

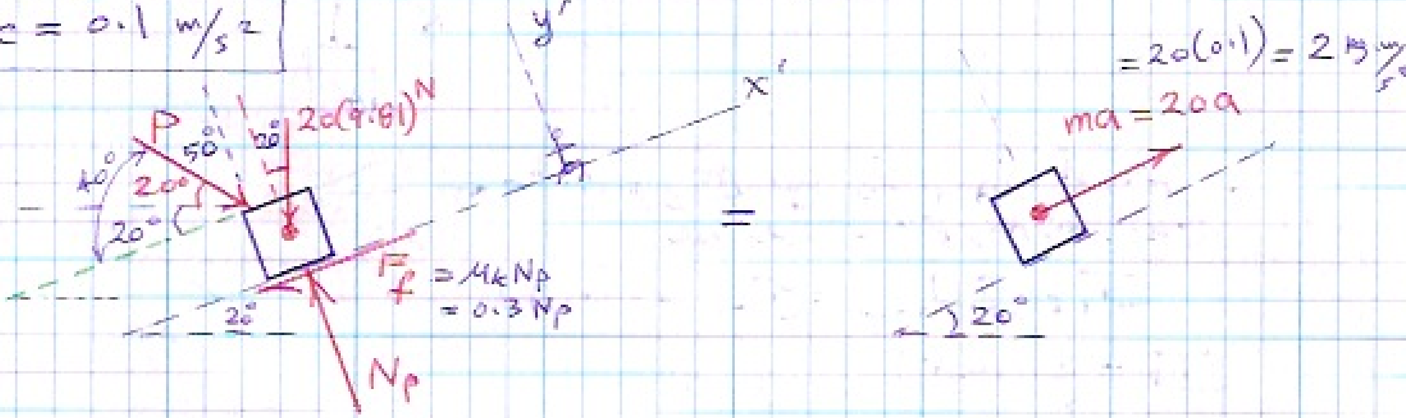
Given $m = 20\,kg$

@ $t = 0\,s$, rest, $t = 10\,s$, $\Delta S = 5\,m$ ↗

$+\nearrow\ s = s_0 + V_0 t + \frac{1}{2} a_c t^2$

$5 = 0 + 0 + \frac{1}{2} a_c (10)^2 \Rightarrow 5 = \frac{100}{2} a_c\ ;\ 5 = 50 a_c\ ;$ (origin, rest)

$a_c = 0.1\ m/s^2$



check for motion

$+\nwarrow \Sigma F_{y'} = 0\ ;\quad N_P - 20(9.81)\cos 20° - P\cos 50° = 0$

$N_P - P\cos 50° = 184.3677$ — ①

$\nearrow \Sigma F_{x'} = m a_x\ ;\quad P\cos 40° - 20(9.81)\sin 20° - 0.3 N_P = 2$

$-0.3 N_P + P\cos 40° = 67.1044$ — ②

solve eqs. ① & ② :

$N_P = 321.6418\ N$

$P = 213.5607\ N$ Ans.

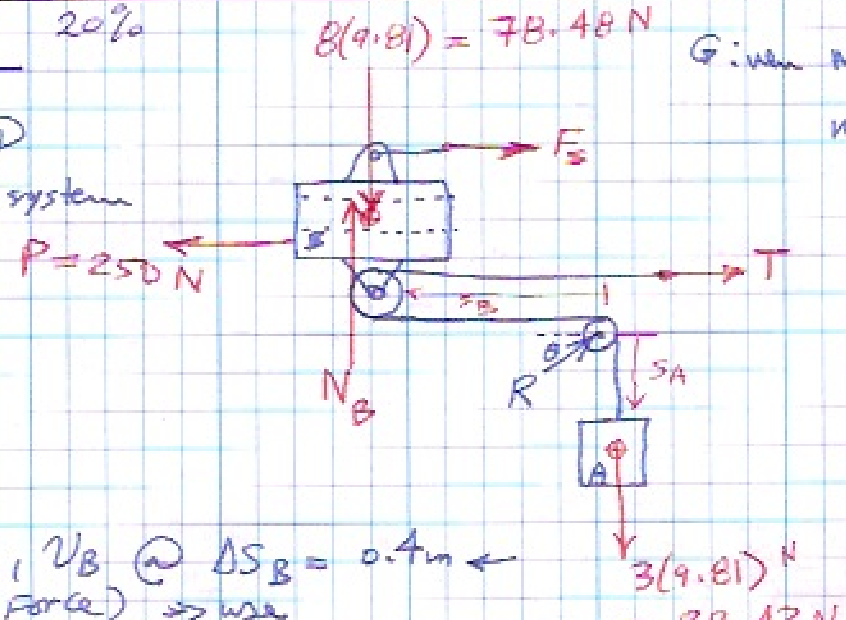
PROBLEM 4: 20%

Given $m_A = 3 kg$

$m_B = 8 kg$

1- Draw F.B.D

A & B as one system



2- Find V_A, V_B @ $\Delta S_B = 0.4 m \leftarrow$
(vel., displ., Force) ⇒ use

$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$ ---- (1)

$\Sigma T_1 = (0$ rest$)$

$\Sigma T_2 = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$

$= \frac{1}{2}(3) V_A^2 + \frac{1}{2} 8 V_B^2 = 1.5 V_A^2 + 4 V_B^2$

Kinematics: $S_A + 2S_B$ = constant; $V_A + 2V_B = 0$; $V_A = -2V_B$

∴ $\Sigma T_2 = 1.5(-2V_B)^2 + 4V_B^2 = 10 V_B^2$; $\Delta S_A + 2\Delta S_B = 0$

$\Delta S_A = -2(0.4)$

$\Delta S_A = 0.8 m \uparrow$

$m_A g \Delta S_A$

$\Sigma U_{1-2} = 250^N(0.4m) - 29.43(0.8m)$

$+ (\frac{1}{2} k S_1^2 - \frac{1}{2} k S_2^2)$ (0 unstretched)

100 N/m

$= 100 - 23.544 + -\frac{1}{2}(250^{N/m})(0.4)^2$

$= 100 - 23.544 - 20$ (8) $= 68.456 J$

$\Sigma U_{1-2} = 56.456 J$

substitute in eqn. (1): $0 + 56.456^J = 10 V_B^2$ (68.456)

$V_B^2 = 5.6456$; $V_B = 2.376 m/s$ Ans.

6.8456

$V_B = 2.6164 m/s$

$V_A = -2(2.376) = 4.752 m/s \uparrow$ Ans. (5.2328)

~~2.6164 m/s~~